Name and Surname : File ....

Grade/Class : 12/......... **Mathematics Teacher** : ....

Hudson Park High School

GRADE 12

MATHEMATICS

Paper 2

Marks : ____ / 150

Time : 3 hours

Examiner : SLT

Date : 6 June 2016

Moderator(s) : SLK

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. **NB**
   - **Start each QUESTION at the *top of a page*.**
   - **Leave *2 lines* open between each of your answers.**
4. **NB** **Fill in the details requested on the front of this Question Paper and hand in your submission in the following manner :**
   - **Question Paper (on top)**
   - **Answer Booklet (below)**

   Do not staple the Question Paper and Answer Booklet together.
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

QUESTION 1 [8 marks]

1. A group of six Grade 12 Mathematics pupils was asked to blow up a balloon each and the volume of the balloon was recorded. Their Mathematics Standardised Test result for Standardised Test 1 is also known :

Standardised Test 1 result % (x)	Volume of the balloon cm^3 (y)
55	4 100
40	4 800
75	5 000
80	6 000
65	3 800
90	4 000

For this data :

1.1. Calculate the

1.1.1. equation of the line of best fit 3

1.1.2. correlation coefficient 1 (4)

1.2. Use (1.1.2) to comment on the trend in the data. (1)

1.3. A pupil who achieved 85 % for Standardised Test 1 was absent on the day the pupils were asked to blow up their balloons.

1.3.1. Had this pupil been at school, predict the volume to which they would have blown up their balloon. 1

1.3.2. How reliable is the prediction in (1.3.1.) ? Explain your answer in detail.

2 (3)

QUESTION 2 [2 marks]

2. For a certain list of data values, where no values are repeated, the following details are known :

$$\sum_{k=1}^{35} x_k = 700$$

$$\sum_{k=1}^{35} (x_k - 20)^2 = 140$$

For the list of data values, calculate the :

2.1. mean (1)

2.2. variance (1)

QUESTION 3 [11 marks]

3. Given :

Data value x	Frequency f
0	2
1	15
2	22
3	37
4	30
5	20
6	13
7	10
8	8
9	5
10	3

For the given data :

3.1. Calculate the :

3.1.1. number of data values 1

3.1.2. mean 1

3.1.3. standard deviation 1

3.1.4. number of data values that lie with 0,45 standard deviations of the mean 2

3.1.5.1. position of the median, and hence the 1

3.1.5.2. value of the median 1 2

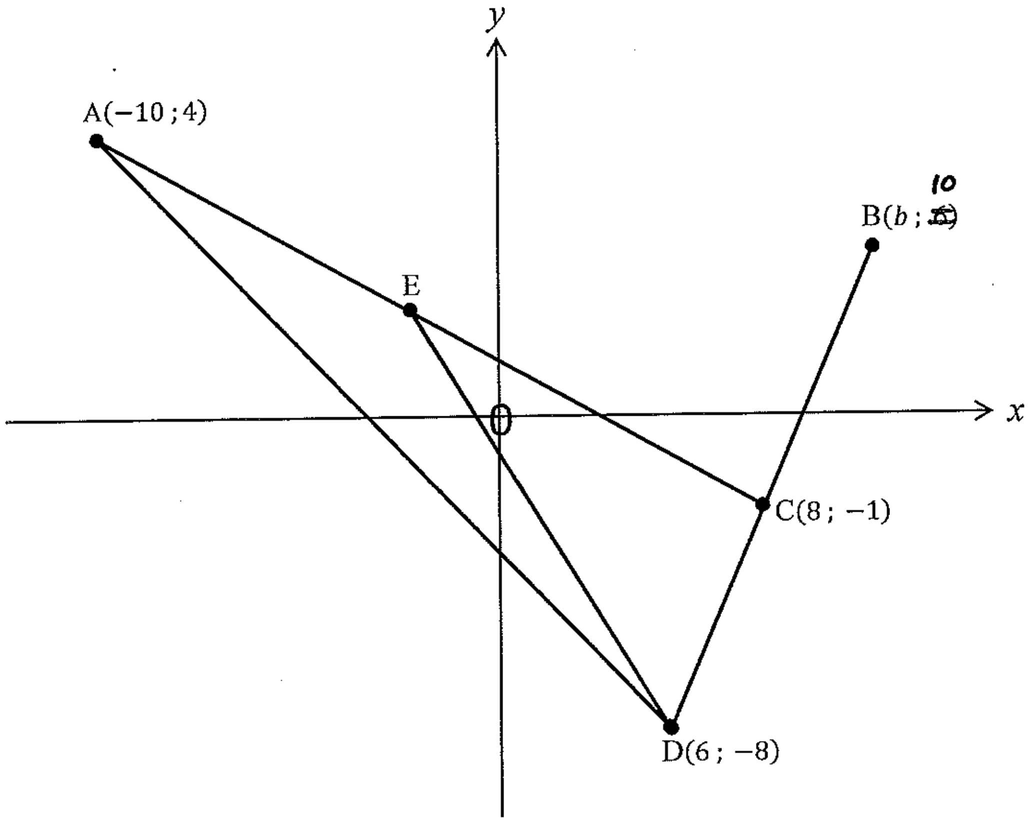
3.1.6. position of the 4th decile (leave your answer as a decimal, if necessary) 1

3.1.7. position of the upper quartile (leave your answer as a decimal, if necessary) 1 (9)

3.2. Use (3.1.2) and (3.1.5.2.) to comment on the skewness of the data. (2)

QUESTION 4 [14 marks]

4. DE is a median of Δ ADC. $A(-10\,;4)$, $C(8\,;\,-1)$, $D(6\,;\,-8)$, $B(b\,;10)$ and $F(f\,;\,-5)$. F is not shown.



Calculate the :

4.1. coordinates of E (2)

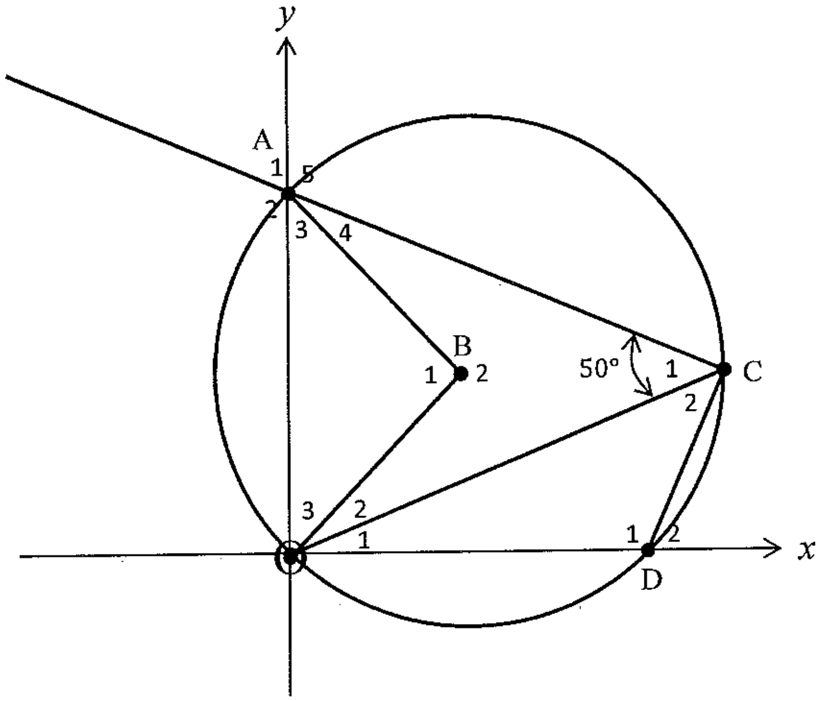
4.2. value of b, if D, C and B are collinear (4)

4.3. value of f, if CF = CD (6)

4.4. coordinates of K, if ACDK is a parallelogram (2)

QUESTION 5 [13 marks]

5. B is the centre of the circle passing through points A, O, D and C. The radius of the circle is 5 cm. $\hat{C}_1 = 50°$ and the equation of line CD is $2y - 3x = -5$.



Calculate :

5.1. the area of Δ AOB (4)

5.2. $\hat{D}_2$ (2)

5.3. $\hat{A}_2$ (3)

5.4. $\hat{O}_{1+2}$ (3)

5.5. the x-coordinate of B, x_B (1)

QUESTION 6 [32 marks]

6.1. If $\cos 18° = m$ (where $0 < m < 1$), determine the following **WITHOUT THE USE OF A CALCULATOR** :

6.1.1. $\cos(-18°)$ 1

6.1.2. $\tan 108°$ 4

6.1.3. $\cos 2208°$ 4

6.1.4. $\sin 9°$ 2 (11)

6.2. Given : $\dfrac{\sin 2x + 1}{\cos 2x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

6.2.1. Prove the given identity. 5

6.2.2. For which value(s) of x will the given identity not be valid ? 2 (7)

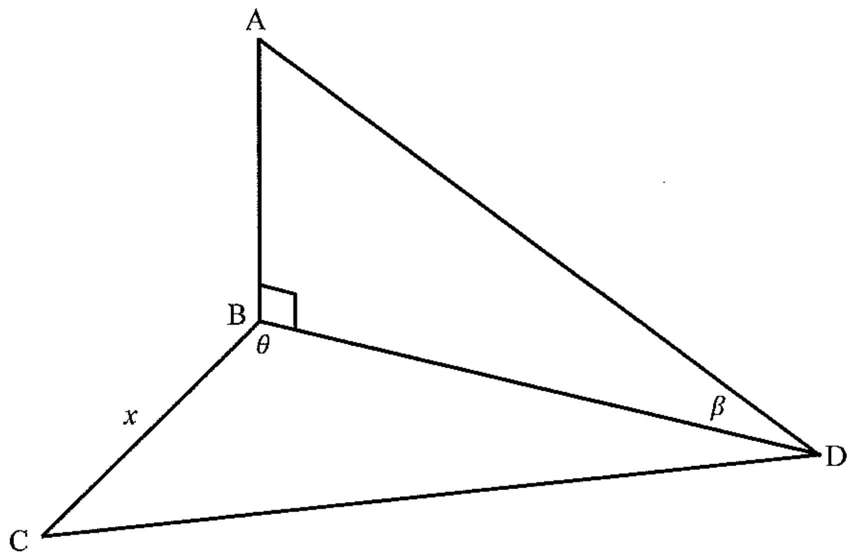
6.3. Simplify fully, **WITHOUT THE USE OF A CALCULATOR** :

$$\frac{\cos 170° \cos 30° + \cos 280° \sin 30°}{\sin 25° \cos 25°}$$ (6)

6.4. Solve for x : $3\cos 2x = 1 + 5\cos x$ (8)

QUESTION 7 [7 marks]

7. B, C and D are in the same horizontal plane. AB is a vertical tower. $CB = CD = x$, $C\hat{B}D = \theta$ and the angle of elevation of A from D is β.



7.1. Show that $BD = 2x\cos\theta$ (5)

7.2. Now determine an expression for AB in terms of x, θ and β. Simplify your answer fully. (2)

QUESTION 8 [14 marks]

8. Given : $f(x) = -\sin x$ and $g(x) = \cos x + 1$

8.1. Sketch the graphs of f and g on the same set of axes, showing all relevant details. (6)

8.2. Use your graphs to determine the value(s) of x for which :

8.2.1. f is decreasing 1

8.2.2. $\cos x + 1 + \sin x = 2$ 2

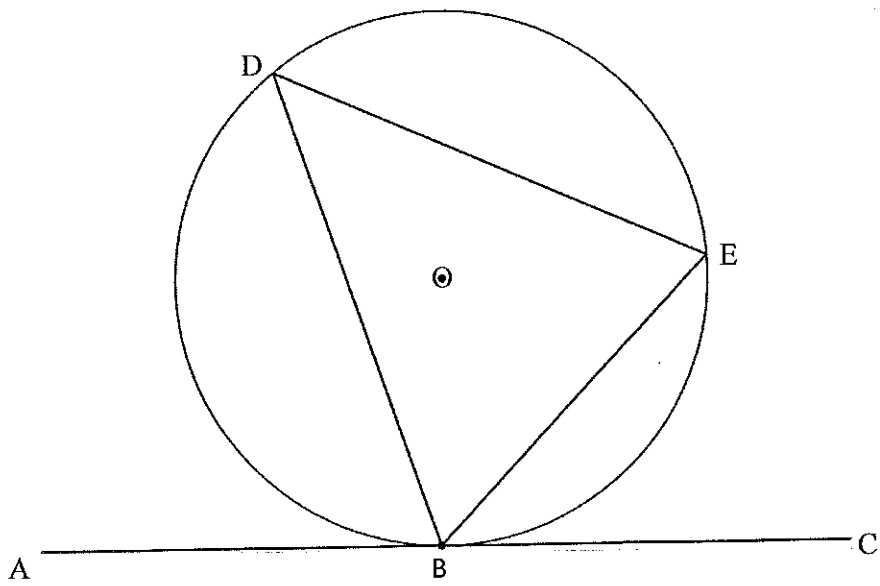
8.2.3. $(-\sin x)(\cos x + 1) \geq 0$ 2

8.2.4. $\cos x + 1 + \sin x > 0$ 1 (6)

8.3. What is the maximum value of : $2\sin^2 x - \sin x + 2\cos^2 x$ (2)

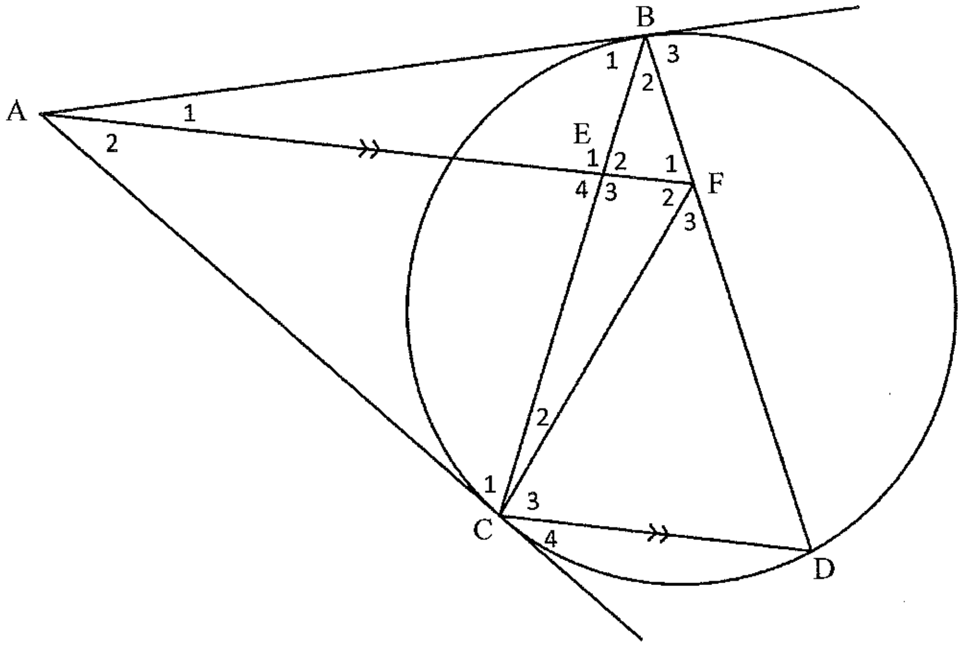
QUESTION 9 [13 marks]

9.1. O is the centre of the circle and ABC is a tangent.



Prove the theorem which states that : $A\hat{B}D = D\hat{E}B$ (4)

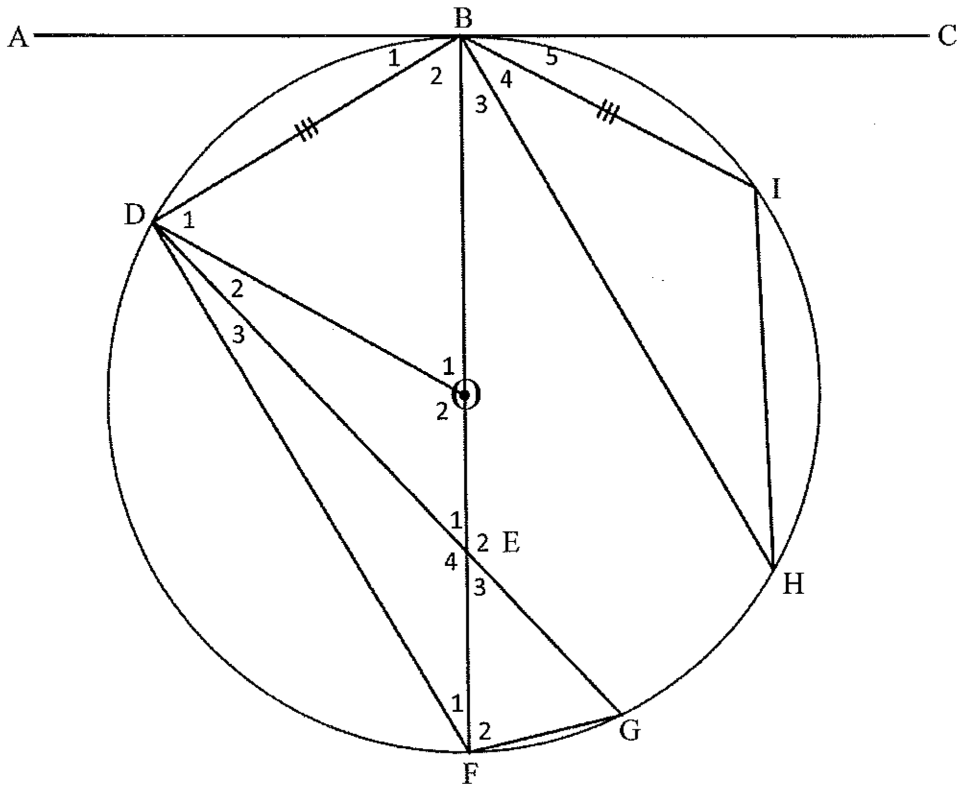
9.2. AB and AC are tangents to the circle, AF // CD and $\hat{B}_1 = 40°$:



9.2.1.	Calculate $\hat{F}_1$	3	
9.2.2.	Prove that ABFC is a cyclic quadrilateral	4	
9.2.3.	Prove that AF bisects $B\hat{F}C$	2	(9)

QUESTION 10 [8 marks]

10. ABC is a tangent to the circle with centre O, $\hat{B}_1 = 25°$ and DB = IB :



Calculate :

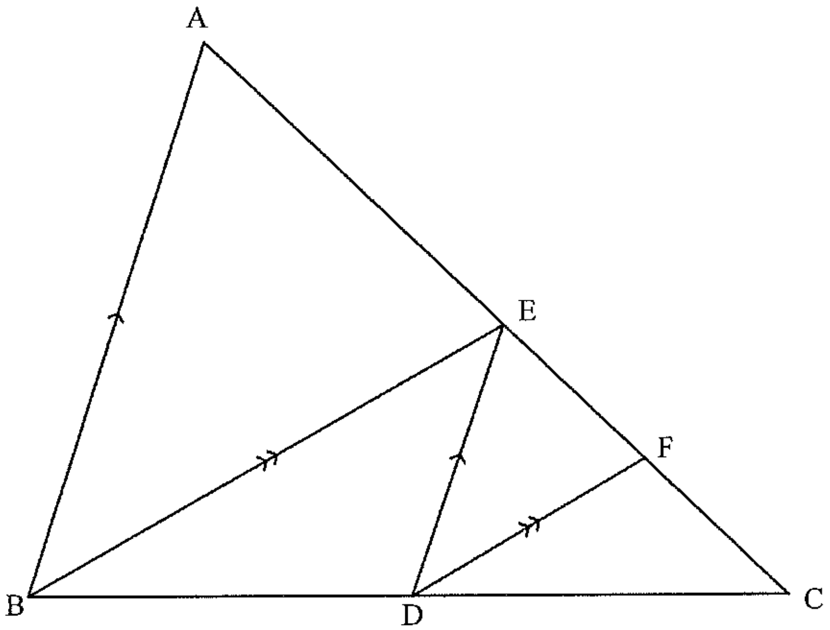
10.1. $\hat{O}_1$ (4)

10.2. $\hat{G}$ (2)

10.3. $\hat{H}$ (2)

QUESTION 11 [4 marks]

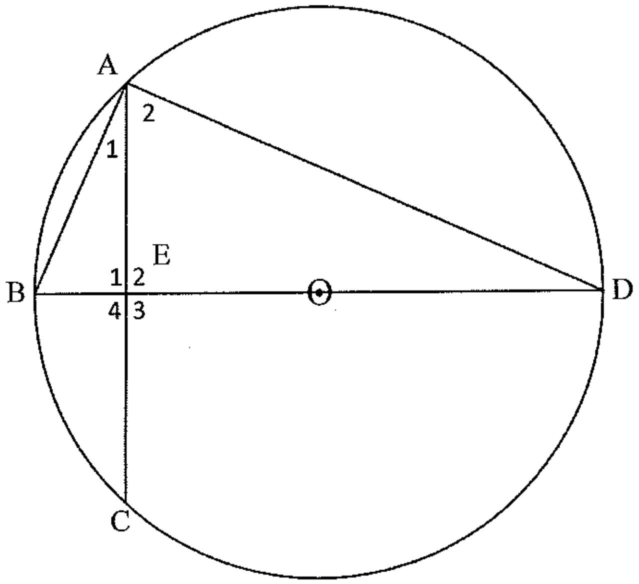
11. BC : CD = 7 : 2 , AB // DE and BE // DF :



Calculate : $\frac{AE}{FC}$ (4)

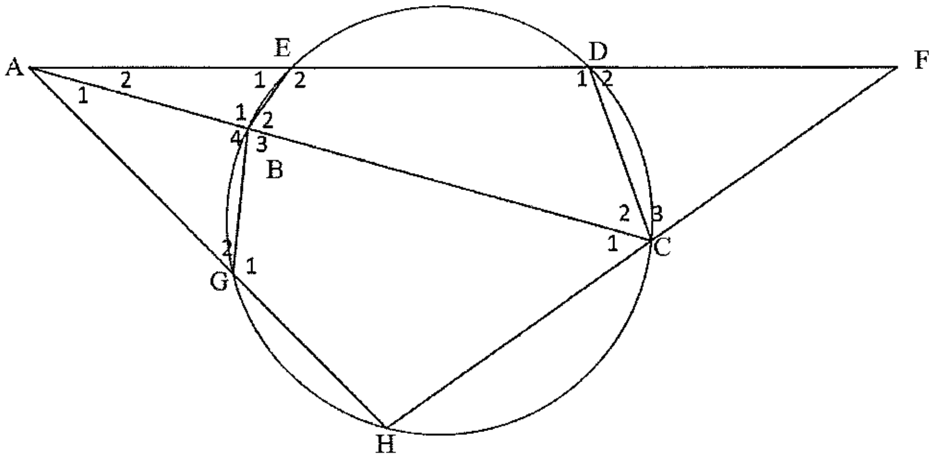
QUESTION 12 [24 marks]

12.1. AE = EC , O is the centre of the circle whose radius is r , OE = 2 and AB = $\sqrt{6}$:



12.1.1.	Prove that Δ ABE /// Δ DBA	5	
12.1.2.	State the length of BE in terms of r	1	
12.1.3.	Calculate the value of r	5	(11)

12.2. BC = AE = 3 , AB = 2 and EF = 4,5 :



Prove that :

12.2.1. BE // CF 3

12.2.2. $\frac{AB}{AC} = \frac{BE}{CF}$ 4

12.2.3. $\frac{AH \cdot BG}{HC} = \frac{BE \cdot AC}{CF}$ 6 (13)

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad$ $A=P(1-ni)$ $\quad$ $A=P(1-i)^n$ $\quad$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad$ $S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $\quad$ $S_n=\frac{a(r^n-1)}{r-1}\ ;r\neq 1$ $\quad$ $S_\infty=\frac{a}{1-r}\ ;\ -1<r<1$

$F=\frac{x[(1+i)^n-1]}{i}$ $\quad$ $P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad$ $y-y_1=m(x-x_1)$ $\quad$ $m=\frac{y_2-y_1}{x_2-x_1}$ $\quad$ $m=\tan\theta$

$$(x-a)^2+(y-b)^2=r^2$$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$a^2=b^2+c^2-2bc.\cos A$

$area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$ $\quad$ $\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\quad$ $\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad$ $P(A\text{ or }B)=P(A)+P(B)-P(A\text{ and }B)$

$\hat{y}=a+bx$ $\quad$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$